For probability calculations I am using:

- a) The table in the back of the book
- b) The probability calculator

(circle one)

Instructions:

1. You may use your calculator, but show your work.
2. Box in your final answers.
3. Label your work, ESPECIALLY your answers.
4. Show your work for partial credit.
5. Raise your hand if you have a question.
6. If you get stuck, move on to the next problem and come back.

<table>
<thead>
<tr>
<th>Page</th>
<th>Points Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
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<td>3</td>
<td>9</td>
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<td>5</td>
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<tr>
<td>6</td>
<td>8</td>
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<tr>
<td>Total</td>
<td>40</td>
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</tbody>
</table>
1) An experiment is run in which 6 observers study 20 low frequency words (like 'ubiquitous') and 20 high frequency words (like 'banana'). Each subject studies both low frequency and high frequency words. The subjects are then asked to recall as many of the 40 words as they can. The experimenter measures the number of each kind of words each subject recalled, which is listed below. Based on prior research, we expect low frequency words to be better remembered, so this will be a directional test.

<table>
<thead>
<tr>
<th>Subject #</th>
<th>Low Frequency Words</th>
<th>High Frequency Words</th>
<th>D</th>
<th>D²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>18</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>11</td>
<td>+1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>19</td>
<td>+1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>16</td>
<td>-4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>8</td>
<td>-3</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
<td>14</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ D = \frac{-3}{6}, \quad \sum D^2 = 30 \]

a) State the null and alternative hypotheses in words and in symbols (i.e. what do you expect the population means to be under each hypothesis?). (1 point)

\[ H_0: \mu_1 \leq \mu_2 \quad \text{How freq. words are not remembered more than high frequency.} \]

\[ H_1: \mu_1 > \mu_2 \quad \text{How freq. words are remembered more than high freq. words} \]

b) Draw a picture of how your summary score is distributed under the null hypothesis, and shade in the critical region(s). Also provide your critical t-score(s). Use \( \alpha = 0.01 \) and perform a directional test that looks to see whether low frequency words are better recalled. (1 point)

\[ df = 5 \quad t_{\text{crit}} = 3.365 \]

[Diagram showing a normal distribution with shaded critical region and t-critical value 3.365]

c) Analyze your data and compute an obtained t-value. Find the following values along the way: (3 points)

i) Sample variance for use in the t-test:

\[ s^2 = \frac{35 - (-5)^2}{6} = 30.8 \]

ii) Estimated standard error:

\[ s_0 = \sqrt{\frac{6.16}{6}} = 1.02 \]

iii) Obtained t-value:

\[ t = \frac{-3.365 - 0}{1.02} = -3.23 \]

d) Numerically compare your obtained and critical t-values and state whether you reject or fail to reject the null hypothesis. Then make a statement about whether different types of words are remembered differently. (2 points)

\[ -3.23 < 3.365 \quad \text{Fail to Reject } H_0 \]

* How freq. words are not remembered more than high freq. words
e) Construct 95% confidence intervals around your mean used in your t-test. (2 points)

\[ c_{\text{crit (5)}}_{\text{2-tailed, .05}} = 2.571 \]

\[ CI: \overline{X} \pm (c_{\text{crit}})(\sigma) \]

\[ CI: -0.83 \pm 1.02 (2.571) \]

\[ -0.83 \pm 2.62 [-3.85, 1.79] \text{ or } [-1.79, 3.85] \]

2) List two problems that can occur with related samples (also called within-subject) designs. (2 points).

Drug washout effects
Practice
Fatigue

3) What is a benefit of related samples (or within-subject) designs over independent samples (or between-subject) designs in terms of the experimental power as dictated by the size of the standard error of the mean? (1 point).

Differences across subjects are removed

4) An experimenter does a repeated sample experiment. She computes a 90% confidence interval around the mean difference between the two conditions. After checking her results, however, she discovered that her standard error of the mean difference was incorrect. She recomputes it, and finds that it is larger than before. Does her 90% confidence interval get larger or smaller as a result of fixing this error, and why? (2 points).

Larger, because

\[ CI: \overline{X} \pm c_{\text{crit}}(\sigma) \]

Interval gets wider
\( \sigma \) gets larger

5) Consider an experiment with 1 factor that has 3 levels. You conduct an ANOVA and find that MSB is larger than zero, but MSW is zero. In the table below, make up data that satisfies this condition. You can use any numbers you like (2 points)

<table>
<thead>
<tr>
<th>Subject 1</th>
<th>Subject 2</th>
<th>Subject 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment 1</td>
<td>Treatment 2</td>
<td>Treatment 3</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>7</td>
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<td>4</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>( \bar{X} )</td>
<td>( \bar{X} )</td>
<td>( \bar{X} )</td>
</tr>
</tbody>
</table>
6) An experiment is conducted in order to determine whether different types of gasoline produce better gas mileage in a certain SUV. In order to test this, 20 identical cars are obtained from the car company and separated into four groups. The cars in each group are given a particular type of gasoline and the gas mileage is measured.

<table>
<thead>
<tr>
<th>Type of Gasoline</th>
<th>Group 1: Regular</th>
<th>Group 2: Mid-Grade</th>
<th>Group 3: Super-Grade</th>
<th>Group 4: Gasohol</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7</td>
<td>6</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

\[
T_j = 25 \quad 20 \quad 30 \quad 45 \\
\eta_j = 5 \quad 5 \quad 5 \quad 5 \\
\bar{X}_j = 5 \quad 4 \quad 6 \quad 9 \\
\Sigma x_j^2 = 147 \quad 106 \quad 196 \quad 411 \\
SS_j = 22 \quad 26 \quad 16 \quad 6
\]

\(N= 20\)  
\(G = 120\)  
\(\Sigma x^2 = 860\)

a) Fill in the missing values for \(SS_1\) and \(\bar{X}_4\). (2 points)

\[
SS_1 = 147 - (22/5) = 22
\]

\[
\bar{X}_4 = 45/5 = 9
\]

b) Perform a standard ANOVA. Use the 0.05 level. Do different types of gasoline affect gas mileage? Follow the steps below.

i) State the null and alternative hypotheses in words and symbols (1 point)

\(H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 \) There is no difference among the different types of gasoline.

\(H_1: \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4 \) There is at least one difference among the means.
ii) Compute \( df_{\text{within}} \) and \( df_{\text{between}} \) and find your Crit F. (2 point)

\[
df_{\text{within}} = N - K = 16 \\
\text{or} \quad k(n - 1) = 16
\]

\[
df_{\text{between}} = k - 1 = 3
\]

iii) Compute SSB (2 points)

\[
SSB = \left( \frac{25^2}{5} + \frac{20^2}{5} + \frac{30^2}{5} + \frac{45^2}{5} \right) - \frac{120^2}{20} \\
= (125 + 80 + 180 + 405) - 720 \\
= 70
\]

iv) Compute SSW (2 points)

\[
SSW = 22 + 26 + 16 + 6 = 70
\]

v) Organize your results into the table below and compute your obtained F value. (6 points)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>Crit F</th>
</tr>
</thead>
<tbody>
<tr>
<td>BETWEEN</td>
<td>70</td>
<td>3</td>
<td>23.3</td>
<td>5.32</td>
<td>3.24</td>
</tr>
<tr>
<td>WITHIN</td>
<td>70</td>
<td>16</td>
<td>4.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>140</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

vi) Do you reject or fail to reject \( H_0 \)? What do you conclude about the effects of different types of gasoline on gas mileage? (1 point)

\[
F(3,16) = 5.32 > F_{\text{crit}} = 3.24
\]

* Reject \( H_0 \), A difference does exist among the 4 types of gasoline.
C) Perform a Scheffe post-hoc test to determine which (if any) means differ from each other in the ANOVA above. Perform only the first two tests, even though more may (or may not) be required.

i) First Test: (4 points)

Test means of Groups 2 and 4.

State which means you are testing and the null and alternative hypotheses for this test:

\[ H_0: \mu_4 = \mu_2 \quad \text{No difference between groups 2 and 4.} \]

\[ H_1: \mu_4 \neq \mu_2 \quad \text{There is a difference between groups 2 and 4.} \]

\[
SSB = \left( \frac{45^2}{5} + \frac{20^2}{5} \right) - \frac{60^2}{10} = (405 + 80) - 432.5 = 62.5
\]

\[
MSB = \frac{62.5}{3} = \frac{SSB}{df_B} = 20.8
\]

Obtained F =

\[
\frac{MSB}{MSW} = \frac{20.8}{4.38} = 4.76
\]

State your critical F value and your conclusions about whether the two means differ:

\[ F_{\text{crit}} = 3.24 < F_{\text{obt}} = 4.76 \quad \star \text{Reject } H_0, \text{ There is a difference between Groups 2 and 4.} \]

ii) Second Test: (4 points)

Test means of Groups 1 and 4.

State the means you are testing and the null and alternative hypotheses for this test:

\[ H_0: \mu_4 = \mu_1 \quad \text{No difference} \]

\[ H_1: \mu_4 \neq \mu_1 \quad \text{There is a difference.} \]

\[
SSB = \left( \frac{45^2}{5} + \frac{25^2}{5} \right) - \frac{70^2}{10} = (405 + 125) - 490 = 40
\]

\[
MSB = \frac{40}{3} = 13.3
\]

Obtained F =

\[
F = \frac{13.3}{4.38} = 3.04
\]

State your critical F value and your conclusions about whether the two means differ:

\[ F_{\text{crit}} = 3.24 > F_{\text{obt}} = 3.04 \quad \star \text{Fail to Reject } H_0, \text{ There is NO difference between 1 and 4.} \]