Communities and Populations

- Two models of population change
  - The logistic map
  - The Lotke-Volterra equations for oscillations in populations
- Prisoner’s dilemma
  - Single play
  - Iterated play
  - Community-wide play
The Logistic Map Function (Robert May)

- Population\(_{T+1}\) = \(r(\text{Population}_T)(1-\text{Population}_T)\)
  - (Population\(_T\)) because next generation’s population based on current population
  - Population in \([0,1]\) range (0 - 100% of carrying capacity)
  - (1-Population\(_T\)) because population decreases as it approaches the carrying capacity
  - \(r\) = fecundity rate

- Three dynamics depending on \(r\)
  - Fixed: Population approaches a stable value
    - 0<\(r<1\): Population becomes extinct
    - 1<\(r<3\): fixed population gradually increases with \(r\)
  - Periodic: Population alternates between 2 or more fixed values
  - Chaotic: Population eventually visits every neighbor in the range \([0,1]\)
The Logistic Map Function

Increasing $r$: Increases maximum achievable population $p_{t+1}$ (obtained if $p_t = 0.5$)
The Logistic Map Function
Bifurcation Diagram
Plot of all population sizes found as a specific initial population changes according to $r$

Increasing $r$ causes period doubling, then chaos

Li & Yorke (1975): “Period 3 implies chaos”: Any sequence with a period of three will display regular cycles of every other period as well as exhibiting chaotic cycles.
Bifurcation Diagram

r<0: extinction

1<r<3: fixed point

r=3: first bifurcation into Period 2. r=3.5: second bifurcation

r=3.57: onset of chaos

r>4: population “escapes” to infinity usually

But, infinite number of points that remain in [0,1] range
These points form a structure very much like the Cantor set
Characterizing Chaotic Dynamics

• **Deterministic yet unpredictable**
  - Non-predictable: No shorthand technique for calculating \( f(f(f(...(x)))) \) other than iteratively applying \( f(x) \)

• **Trajectories are dense**
  - Trajectories will pass arbitrarily close to any point in space
  - Two initial conditions always intrude into each other’s territory

• **Sensitivity to initial conditions**
  - Small changes in initial conditions become magnified quickly

• **Dense periodic trajectories**
  - There are an infinite number of periodic states that are arbitrarily close to any other state
  - But, chances of falling in these states approaches zero

Sensitivity to initial conditions
**Two Species Populations**

- **Predator-prey systems**
  - Prey increase numbers of predators
  - Predators decrease numbers of prey
  - Kind of activator-inhibitor system

- **Lotka-Volterra equations**
  - Coupled differential equations for changes in predator and prey populations
  - Population oscillations are generally predicted
  - Oscillations are sometimes, but not always found in nature

\[
\begin{align*}
\frac{\partial G}{\partial t} &= aG - bGR, \\
\frac{\partial R}{\partial t} &= dbGR - cR
\end{align*}
\]

G = Grass population, R = Rabbit population
a = rate of grass growth without predation
b = consumption of grass by rabbits per encounter
c = death rate of rabbits
d = efficiency of turning grass into rabbits
Lotka-Voterra Systems

- equilibria points exist, but are unstable
  - $aG - bGR = \text{rabbit rate of change, so no change if } R = a/b$
  - $dbGR - cR = \text{grass rate of change, so no change if } G = db/c$
  - Equilibrium point: $f(x) = x$
  - Stable point: small perturbation from point returns system to point

![Low initial prey population](image1)

![High initial prey population](image2)

Phase portrait
Oscillations in Natural Populations

Huffaker’s (1958) populations of two mites
Problems with the Lotka-Volterra System

• Assumes prey spontaneously grow without substrate
• No carrying capacity of environment (growth without limit)
• No notion of space: All predators and prey are at exactly the same point and thus have access to each other
  – If incorporate space, population dynamics are markedly different
  – Predators and Prey coexist seven times longer when spatial movement is limited (Huffaker, 1958)
  – Population waves through space
  – Chemical analog: the BZ reaction
  – Standing spatial waves of population if predators move faster than prey! (Hassell, May, & Comins, 1999)
The BZ Chemical Reaction
The Prisoner’s Dilemma

• Conflict between individual rationality and overall utility
• Social dilemmas: How to achieve a good solution for the group when individual self-interest, if pursued, would lead to a poor solution
  – Tragedy of the commons
  – Public radio stations
  – Nuclear weapons build-up
• Tension between cooperation and competition
  – How can altruism be perpetuated in a group?
    • Applied issue: how to increase cooperation rates. Persuasion
    • Traditional claim: Only individual, not group-level, selection (Hamilton, 1964; Williams, 1966)
  – Group identification in social psychology: “We” not only “I” is important
The Prisoner’s Dilemma (PD)

<table>
<thead>
<tr>
<th></th>
<th>Prisoner 1</th>
<th>Prisoner 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cooperate</strong></td>
<td>1 year</td>
<td>1 year</td>
</tr>
<tr>
<td><strong>Defect</strong></td>
<td>Free</td>
<td>5 years</td>
</tr>
<tr>
<td><strong>Defect</strong></td>
<td>5 years</td>
<td>3 years</td>
</tr>
<tr>
<td><strong>Free</strong></td>
<td>3 years</td>
<td>3 years</td>
</tr>
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</table>

Each player maximizes their outcome by defecting, but the overall payoff if both players defect is poor.
Treatments of the PD

- Standard game theory: defection as rational in single-play PD
- Hofstadter: cooperation rational even in single-play PD
- Multi-play PD
  - No “best play” strategy - depends on partners
  - PD competition (Axelrod, 1980)
    - Round-robin tournament of programs partnering each other
      - Can’t choose who you play against
    - Tit-for-tat performed best: Start off cooperating, then choose whatever opponent chose in last round
      - Enables lasting cooperation, but is not victimized by defectors
      - Never outperforms partner, but does better overall than any of them
      - Performs worse if randomness included - not forgiving enough. For these environments, generous tit-for-tat outperforms tit-for-tat (Nowak & Sigmund, 1993)
      - Tit-for-tat and similar strategies evolve with genetic algorithm
  - Pavlov outperforms even generous tit-for-tat in probabilistic environments (Nowak & Sigmund, 1993)
    - Keep strategy if good outcome previously, shift if poor outcome
## Strategies in Multi-play PD

<table>
<thead>
<tr>
<th>Last move</th>
<th>Opponent’s last move</th>
<th>Outcome</th>
<th>Next move</th>
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<tr>
<td>C</td>
<td>C</td>
<td>3</td>
<td>C</td>
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<tr>
<td>C</td>
<td>D</td>
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<tr>
<td>D</td>
<td>C</td>
<td>5</td>
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<tr>
<td>D</td>
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<td>1</td>
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</tr>
<tr>
<td>D</td>
<td>D</td>
<td>1</td>
<td>C</td>
</tr>
</tbody>
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**Tit-For-Tat**

**Pavlov**
### Strategies in Multi-play PD

<table>
<thead>
<tr>
<th>I cooperated</th>
<th>I cooperated</th>
<th>I defected</th>
<th>I defected</th>
<th>Code</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>They cooperated</td>
<td>They defected</td>
<td>They cooperated</td>
<td>They defected</td>
<td>CCCC</td>
<td>Always cooperate</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>C</td>
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<td>Always cooperate</td>
</tr>
<tr>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>DDDD</td>
<td>Always defect</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
<td>C</td>
<td>D</td>
<td>CDCD</td>
<td>Tit-for-tat</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Do whatever was done to me last</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
<td>D</td>
<td>C</td>
<td>CDDC</td>
<td>&quot;Pavlov&quot; - If I had a bad outcome, change strategy</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>CDDD</td>
<td>Spiteful - once anybody defects, it continues to defect</td>
</tr>
<tr>
<td>D</td>
<td>D</td>
<td>C</td>
<td>C</td>
<td>DDCC</td>
<td>Waffle - alternates between cooperation and defection</td>
</tr>
</tbody>
</table>
Spatial PD (Nowak & May, 1992)

- Agents play PD with their spatially determined neighbors
- No memory for previous plays: only cooperators and defectors
- Rules
  - Play PD games with 8 neighbors
  - Player adopts strategy with highest outcome of its 9 neighbors (including itself)
  - Cooperation when partner cooperates -> 1 point
  - Any time partner defects -> 0 points
  - Defection of partner cooperates -> d points

gray = defector
white = cooperator
Spatial PD (Nowak & May, 1992)

d=1.75: filaments of defectors around edges of cooperators
d=1.9: Single defector in middle. Defector takes advantage of cooperators, but clusters of cooperators do better than sets of defectors.