The Dynamics of Similarity

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Similarity depends on representations of stimuli that are constructed and changed during comparison-making. Specific features may be selectively weighted during comparison, and the features used in a comparison may themselves be a product of the comparison process. Traditional models of similarity and analogy rely on representations that are assumed to exist prior to comparison and are inflexible. Evidence from previous research indicates that weighting of features in similarity judgments may vary dynamically during processing (Goldstone, 1994; Goldstone & Medin, 1994). SIAM (Goldstone, 1994), a model providing an account of dynamic weighting, is discussed. Additional studies indicate that features may be developed or introduced during similarity judgments. A methodology for examining process-oriented models that may account for flexible representations is proposed.

Keywords: similarity, analogy, comparison-making, dynamics

The Dynamics of Similarity

Art museum curators are an under-appreciated lot. By simply choosing the location for art within an exhibition, they can both create a tone for the exhibit and help viewers form fresh interpretations of even very familiar pieces. Close placement of two pieces of artwork in an exhibit invites comparisons; facets of the art that would have gone unseen emerge. Some elements are suppressed. A painting presented in one setting may seem entirely different in another. Even in similarity judgments of less rich stimuli, comparisons can lead to the emphasis of particular features, or even to the realization of new features. In this article, we will argue that similarity involves dynamic processes in which representations of the elements under consideration are flexible, change over time, and change in response to context.

By exploring the dynamic nature of similarity, we can better understand the varied roles it plays in cognition. In this paper, we address how people assess similarity, and we discuss traditional models and describe their limitations with respect to flexibility. We then present approaches that correct some of these limitations, and discuss evidence relevant to these particular formal models of similarity. We also provide evidence indicating a need for even greater flexibility in our models of similarity and analogy. We end with discussion of a proposed methodology for studying dynamics in comparison processes.

Traditional Models Theory

Two of the most widely-discussed traditional approaches to similarity are geometric models,
exemplified by multidimensional scaling (MDS) models, and featural models, exemplified by Tversky's (1977) Contrast Model. In geometric models objects are represented as points in a psychological space. The similarity of two entities \( i \) and \( j \) is taken to be inversely related to their distance, \( D(i,j) \), which is computed by:

\[
D(i,j) = \left( \frac{1}{n} \sum_{k=1}^{n} |x_{ik} - x_{jk}|^{1/r} \right)^{r}
\]

where \( n \) is the number of dimensions, \( X_{ik} \) is the value of dimension \( k \) for entity \( i \), and \( r \) is a parameter that allows different spatial metrics to be used. A Euclidean metric \((r = 2)\) often provides good fits to human similarity judgments when the entities are holistically perceived or the underlying dimensions are psychologically fused, whereas a City-block metric \((r = 1)\) often provides a better fit when entities are clearly divisible into separate dimensions (Garner, 1974).

Shepard (1987) has made a compelling case that cognitive assessments of similarity are related by an inverse exponential function to distance in MDS space.

Richardson's (1938) fundamental insight, which is the basis of contemporary use of MDS, was to begin with subjects' judgments of pair-wise object dissimilarity, and work backward to determine the dimensions and dimension values that subjects used in making their judgments. MDS algorithms proceed by placing entities in an \( N \)-dimensional space such that the distances between the entities accurately reflect the empirically observed similarities. For example, if we asked people to rate the similarities [on a scale from 1 (low similarity) to 10 (high similarity)] of Russia, Cuba, and Jamaica, we might find:

- Similarity (Russia, Cuba) = 7
- Similarity (Russia, Jamaica) = 1
- Similarity (Cuba, Jamaica) = 8

In finding an MDS solution, we would try to position the three countries in a space such that countries that are rated as being highly similar are very close to each other in the space. We might find that the interpoint distances suggested by the similarity ratings cannot be simultaneously satisfied in a given dimensional space. If we limit ourselves to a single dimension (we place the countries on a "number line"), then we might not be able to simultaneously place Russia near Cuba (similarity = 7) and place Russia far away from Jamaica (similarity = 1). In MDS terms, the "stress" of the one-dimensional solution would be high. Accordingly, we might position the points in two-dimensional space. Once the points are positioned in a way that faithfully mirrors the ratings obtained, it may be possible to give interpretations to the two dimensions. In this example, dimensions may correspond to "former political affiliation" and "climate." Russia and Cuba would have similar values on the former dimension; Jamaica and Cuba would have similar values on the latter dimension.

MDS models have been successful in expressing cognitive structures in stimulus domains as far removed as animals (Smith, Shoben, & Rips, 1974), Rorschach ink blots (Osterholm, Woods, & Le Ues, 1985), chess positions (Horgan, Millics, & Meiney, 1989), and air flight scenarios (Schvaneveldt, 1985). It would appear that many of the objects, situations, and concepts that we come into contact with are structured in terms of dimensions, and a geometric interpretation of the dimensional organization captures a good deal of that structure.

Standard geometric models assume minimality \([D(A,B) \geq D(A,A) = 0]\), symmetry \([D(A,B) = D(B,A)]\), and the triangle inequality \([D(A,B) + D(B,C) \geq D(A,C)]\). Tversky (1977) criticized geometric models on the grounds that violations of all three assumptions are empirically observed. Minimality may be violated because not all identical object seem equally similar; complex objects that are identical (e.g. twins) can be more similar to each other than simpler identical objects (e.g. two squares). Asymmetrical similarity occurs when an object with many features is judged as less similar to a sparser ob-
ject than vice versa; for example, North Korea is judged to be more like China than China is to North Korea (Tversky, 1977). The triangle inequality can be violated when A (e.g., "Jamaica") and B ("Cuba") share an identical feature (climate), and B ("Cuba") and C ("Russia") share an identical feature (political affiliation), but A and C share no feature in common (Tversky & Gati, 1982). Although geometric models can be modified to correct these assumptions (Nosofsky, 1991), Tversky suggested an alternative featural approach, the Contrast Model, wherein similarity is determined by matching features of compared entities, and integrating these features by the formula

\[ S(A, B) = \theta f(A \cap B) - \alpha f(A - B) - \beta f(B - A). \]

The similarity of A to B, S(A, B), is expressed as a linear combination of the measure of the common and distinctive features. The term \(A \cap B\) represents the features that items A and B have in common. \((A - B)\) represents the features that A has but B does not. \((B - A)\) represents the features that B has but A possesses. The terms \(\theta\), \(\alpha\), and \(\beta\) reflect the weights given to the common and distinctive components, and the function \(f\) is often simply assumed to be additive.

Traditional Models and Dynamic Similarity

Both featural and geometric models assume features are fixed before comparisons. The Contrast Model assumes that preprocessors supply featural descriptions of stimuli that are then used as the inputs to the model. In standard MDS models, once a final configuration is achieved by an MDS routine, the stimuli are represented as fixed points in the space. Extensions to standard MDS models such as the Generalized Context Model (Nosofsky, 1986) perform operations that stretch or shrink dimensions in the space, but do not change the ordinal positions of stimuli relative to each other. In this sense, these models can be said to be static; they do not allow for flexible representations during comparisons.

Both featural and geometric models encounter difficulties when compared items are related hierarchically (an item is contained within another) or propositionally (an item takes another as an argument). A proposition is an assertion about the relation between informational entities (Palmer, 1975). For example, relations in a visual domain might include Above, Near, Right, Inside, and Larger-than. The informational entities might include features, and values along dimensions such as square, black, and 3 cm. Propositional representations are characterized by relational predicates that take arguments. A two-place predicate, such as Above would take two arguments. The order of the arguments in the predicate is critical. The meaning of a proposition changes if the argument order changes except in the case of symmetric relations such as Near-to. For example, Above (Triangle, Circle) does not represent the same fact as Above (Circle, Triangle).

By representing objects as points in a conceptual space, MDS models have difficulty representing hierarchical and propositional relations. Objects are represented by their coordinates; their values on a set of independent dimensions. Each object dimension is orthogonal to the other dimensions, and no dimension is a subset of another dimension. Even if we allow dimensions to be nonorthogonal, no hierarchy of features is implied. Nonorthogonal dimensions in MDS are sometimes used to express contingent relations between two dimensions (i.e. if large objects tend to be circular). This dimensional contingency, however, is not equivalent to a hierarchical nesting of dimensions.

Propositional representations also pose difficulties for MDS and featural accounts of similarity. For example, in Figure 1, the relationships between objects make it difficult to construct a simple featural account of similarity. In this task, participants were asked to decide whether A or B is more similar to C. If we choose A over B as more similar, the panel suggests that the proposition similarity of the two subjects also based on spatial locations and their shapes.

However, it is not clear whether the left-most object of the T panel suggests that the proposition similarity of the two subjects also based on spatial locations and their shapes.

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B is more similar to A than to C, so we choose A over B as more similar. The second panel suggests that the proposition similarity of the two subjects also based on spatial locations and their shapes.

However, it is not clear whether the left-most object of the T panel suggests that the proposition similarity of the two subjects also based on spatial locations and their shapes.
B is more similar to T. The strong tendency to choose A over B as more similar to T in the first panel suggests that the feature “square” influences similarity. Other choices indicated that subjects also based similarity judgments on the spatial locations and shadings of objects as well as their shapes.

However, it is not sufficient to represent the left-most object of T as Left, Square, Black, Right, Circle. White and base similarity on the number of shared and distinctive features. In the second panel, A is again judged to be more similar to T than is B. Both A and B have the features “Black” and “Square.” The only difference is that for A and T, but not B, the “Black” and “Square” features belong to the same object. This is not incompatible with feature set representations as long as we include the possibility of conjunctive features in addition to simple features such as “Black” and “Square” (Gluck, 1991; Hayes-Roth & Hayes-Roth, 1977). By including the conjunctive feature “Black-Square,” that is possessed by T and A, we can explain, using feature sets, why T is more similar to A than B. The third panel demonstrates the need for a “Black-Left” feature, and other data indicates a need for a “Square-Left” feature. Altogether, if we wish to explain the similarity judgments that people make we need a feature set representation that includes six features (three simple and three complex) to represent the square of T. The number of features needed quickly climbs. Feature models are not structural descriptions, and require many features to completely describe a scene organized into parts. Similar problems exist for dimensional accounts of similarity.

A more concise propositional representation of T might be Left-of (In-same-object (black, square), In-same-object (white, circle)). The propositional representation trades off processing complexity for representational efficiency. The representation is efficient in that only seven non-syntactic symbols are required whereas the feature set representation required 24 features to obtain the same representational power. However, the propositional representation assumes that there will be processes that operate on the representation to determine the similarity. The process that operates on the feature set representation is quite simple. Similarity is computed by simply matching identical features between scenes, and increasing similarity as a function of this pool of shared features minus a function of the features that were not matched. For the propositional representation, processes are required that count shared arguments more for similarity if they are in the same order, in the same relation, and combined with the same arguments in the same relation.

The complex, propositional representation requires a more complex set of processes to use it than were required for featural or dimensional representations. Questions like “What is done with matching simple features that are not in the same relation?” arise with propositional, but not feature set, representations. However, it is likely that the added representational and processing complexity of propositional representations is a price worth paying for the ability to efficiently capture the structure of a scene, particularly when the scene has many structured elements. The Similarity as Interactive Activation and Mapping model (SIAM) (Goldstone, 1994) to be described in the next section is an attempt to provide a formal processing account that uses propositional representations in determining similarity.

One particularly important problem that arises when structured scenes are compared concerns establishing correspondences or alignments between scene elements. One may account for the choice of A over B as more similar to T in the second panel of Figure 1 by pointing out that, although both A and B have a black object, B’s black object does not correspond to T’s black object. It is assumed that the similarity of two scenes is increased more by a matching feature that occurs between corresponding, rather than
noncorresponding objects. As the lowest panel indicates, object correspondence may depend on location rather than shape.

There is growing evidence that important aspects of similarity and analogy are not accounted for in featural and geometric models. Recent research demonstrates two such characteristics of similarity: dynamic weighting of the contribution of specific features to similarity judgments, and the dynamic development of features. We will consider evidence that not only is the importance of a feature determined during comparison, but also that a feature may emerge during comparison. We will consider recent theories of similarity and analogy that employ flexible representations and allow for a more process-oriented account of similarity and analogy.

**Dynamic Weighting of Features Theory**

Gentner (1983) proposed the structure-mapping theory of analogy, and posited that analogical reasoning involves the mapping of structured information from one domain into another. Relations between objects in a source analog are compared to relations between objects in a target analog. For example, in making an analogy between the solar system and an atom, the planets circling the sun are mapped to electrons circling the nucleus. Roles of objects, rather than objects themselves, are mapped one-to-one between source and target. Alignment is the process of finding the best correspondences between source and target. Goldstone (1994) showed that alignment affects the influence of features in similarity judgments of structured scenes. The term “scene” will be used to describe any structured entity, including landscapes, stories, or sets of objects. When assessing similarity, observers tend to find correspondences between parts of scenes. Objects are said to be aligned if they correspond to each other. In the previous analogy, planets and electrons are aligned. Planets correspond to electrons; they are both “things that circle some central object.” A match between features associated with properly aligned objects in two scenes is termed a Match In Place (MIP). A match between features that are associated with parts of two scenes that are not well-aligned is termed a Match Out of Place (MOP). For example, in comparing cricket to baseball, the feature white is shared between baseballs and cricket players’ shirts. Since this shared feature occurs between parts that do not correspond to each other, it is considered a MOP node. The uncorrespondence, the Object C of the B-Object A and Objects B and C are considered a MOP.

The primary research on MOPs has been conducted by Rumelhart and his colleagues. Rumelhart's (1979) simulation of word perception is based on several models of the Structure MOP (SMOP). Forbus, a language-processing program, employs structure maps based on structures learned from its environment.

The connectionist model of analogy (ACME) (Holyoak and Thagard, 1989)

SIAM determines whether an object is made of objects or objects of objects. In the diagram, slots that are filled with objects are considered to be objects of objects, while slots that are left empty are considered to be objects.
Sample nodes and connections in SIAM. Each rectangular slot represents a correspondence node. The upper left slot represents the node placing Object A and Object C into correspondence; the slot immediately below it represents the correspondence node for the heads of Object A and Object C. Consistent nodes excite each other, while inconsistent nodes inhibit each other. The node for the Object A-Object C correspondence inhibits the node for Object B-Object C correspondence, since only one object may be mapped to another. Both Object A and Object B may not correspond with Object C.

The primary motivation for the Similarity as Interactive Activation and Mapping model (SIAM) (Goldstone, 1994) was to provide an account of similarity judgments between structured entities. In the Feature Contrast Model and MDS-based models of similarity the alignment of parts of compared items is not considered. SIAM was inspired by McClelland and Rumelhart’s (1981) interactive activation model of word perception, and has commonalities with several models of analogical reasoning. As with the Structure Mapping Engine (SME) (Falkenhainer, Forbus, and Gentner, 1989), SIAM operates on structured representations. SIAM determines correspondences between parts of scenes. The connectionist architecture of SIAM is similar to that of Analogical Constraint Mapping Engine (ACME) (Holyoak and Thagard, 1989).

SIAM determines the similarity of two scenes made of objects; objects are made of feature slots that are filled with particular feature values. Scenes are structured: objects have specific roles within a scene. In comparing scenes, both the role of an object within a scene and the features which make up the object are important. As scenes are compared, objects in one scene are placed in correspondence, or aligned, with objects in the other scene. These correspondences are influenced by perceptual similarity, and by other correspondences—a correspondence that is consistent with other correspondences carries more weight. Correspondences in turn influence the weight that matching elements will have on the similarity judgment.

In the model, nodes represent hypotheses that two objects (object-to-object nodes) or features (feature-to-feature nodes) are in correspondence. Both excitatory and inhibitory activation passes between nodes. For example, in Figure 2, a scene with Objects A and B is compared to a scene with Objects C and D. In this example, all objects have four features implemented as feature slots. A feature slot is a general category of a
feature. A feature slot such as weight can take on values such as "light" or "20 kilograms." It is assumed that any feature values which can fill the same feature slot can be compared. Processing proceeds as parallel constraint satisfaction.

At the beginning of processing, SIAM has no predisposition to place objects in correspondence with each other. Features are the first elements to be placed in correspondence, and contribute to the formulation of object correspondences. Once objects begin to be aligned, these alignments in turn influence the lower-level feature correspondences. Object correspondences influence activation of feature correspondences, and vice-versa. Nodes that are consistent with one another send excitatory activation; nodes that are inconsistent send inhibitory activation. Nodes are inconsistent if they result in two-to-one mappings—for example, when two objects are associated with the same object in the other scene.

SIAM processes a similarity comparison by activation passing. During a time cycle, activation passes between each node; the net input for node \( i \) is determined by:

\[
\text{net}_{i(t)} = \sum_{j=1}^{n} (A_{j(t)} W_{ij}) - \text{MIN} \frac{\text{MAX} - \text{MIN}}{n (\text{MAX} - \text{MIN})}
\]

where \( n \) is the number of all links to node \( i \), \( A_{j(t)} \) is the activation of node \( j \) at time \( t \), and \( W_{ij} \) is the weight of the link from unit \( j \) to unit \( i \). Weights are set to 1.0 for excitatory connections or -1.0 for inhibitory connections. Net_{i(t)} is normalized by MAX and MIN. MAX and MIN are the maximum and minimum values node \( i \) could attain were it not normalized. The activation of node \( i \) is a function of the activation at time \( t \) and the net input at time \( t+1 \); if Net_{i(t)} > 0.5 then

\[
A(t+1) = A(t) + (1 - A(t)) \times (\text{net}_{i(t)} - .5) \times B,
\]

otherwise

\[
A(t+1) = A(t) - A(t) \times (0.5 - \text{net}_{i(t)}) \times B.
\]

where \( B \) is a parameter for the amount of activation adjustment.

Both the similarity of scenes and the alignment of the parts of scenes are determined by the activation patterns. Nodes with high activation will both weigh heavily in the similarity judgment and will tend to be placed in alignment. Similarity is determined by:

\[
\text{Similarity} = \frac{\sum_{i=1}^{n} (\text{matchvalue}_{i} \times A_{i})}{\sum_{i=1}^{n} A_{i}}
\]

where \( n \) is the number of feature-to-feature nodes representing two scenes \( n = FO^2 \) where \( F \) is the number of features in an object, \( O \) is the number of objects in a scene). \( A_{i} \) is the activation of node \( i \). The match value of node \( i \) is between 0 and 1. Nodes activations change with time; match values are determined perceptually by featural similarities between scene elements.

The node activations, \( A_{i} \), also determine the strength of correspondence between scene objects. Mapping accuracy, the ability of subjects to align objects between scenes optimally, is modeled by a ratio choice rule. The probability of making a particular mapping is calculated by dividing the sum of the activations associated with that mapping by the sum of activations of all nodes:

\[
P(m) = 1 - \frac{(1 - A_{i})}{\sum_{d=1}^{C(M)} (1 - A_{d})},
\]

where \( C(M) \) is the set of object-to-object mappings nodes consistent with mapping \( M \), and \( n \) is the number of object-to-object nodes.

**Empirical Support for SIAM**

A series of experiments by Goldstone demonstrate the influence of alignment on similarity (Goldstone, 1994). In a typical experiment, two scenes consisting of two schematic butterflies are shown on the left (Figure 2). One scene was the second was generated from the first; features of the first was generated each change was represented by the transition "XY--XY" represented two Matches In Place (MIPs) swapping features of the butterflies, resulting in three new (MOPs). In Figure 3, MOP: body color of butterfly. The butterfly.

Participants were asked to rate the similarity of the two images. The ratings of the two images were influenced similarity.
Trials consisted of a starting display and a changed display. In these examples of three possible alterations, only body color is considered. No change results in 2 Matches In Place (MIPs). Switching body shadings results in 2 Matches Out Of Place (MOPs). Keeping the body color of only one butterfly and attributing it to another butterfly results in 1 MOP.

Figure 3

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The finding is naturally handled by SIAM's posited interaction between (in)consistent feature-to-feature nodes. Feature-to-feature nodes that consistently place feature values from the same dimension in correspondence send direct activation to one another. Two MOPs, if created by swapping feature values, will support one another, although they are inconsistent with the globally optimal object-to-object correspondences.

SIAM also predicts the nonsignificant effect due to a MOP that competes against a MIP. In scenes where a feature matches both an aligned object and a nonaligned object (1 MOP and 1 MIP), the MOP will not contribute to the similarity rating. Two-to-one mappings are inconsistent; the MOP will be suppressed by the MIP. The MOP will in turn suppress the MIP, but to a much lesser degree.

SIAM correctly predicts an influence of feature distribution on similarity ratings. MIPs that are concentrated in object pairs (several matches exist between two specific butterflies) or in dimensions (for example, several matches exist on the body shading dimension)
increased similarity more than MIPs that are distributed across object pairs and/or dimensions. In SIAM, the dimension-concentrated advantage stems from the influence of feature-to-feature nodes on (in)consistent feature-to-feature nodes. Feature-to-feature nodes for a particular dimension are inconsistent if they create a many-to-one mapping between feature values, otherwise they are consistent. Consistent feature-to-feature nodes mutually excite each other, thus increasing their influence on similarity. The object-concentrated advantage accrues from feature-to-object and object-to-feature connections. Objects with many concentrated feature matches will be placed in strong correspondence, and will feed activation back down to the individual feature matches.

Goldstone & Medin (1994) also demonstrated the dynamic nature of feature weighting, showing that MIPs had increasing influence relative to MOPs as processing progressed. Subjects were asked to decide if two scenes contained the same butterflies within a deadline (time limit) that varied within the experiment. On half the trials, identical butterflies were shown in both scenes; in the other half, one scene was a transformed version of the other scene. Errors on the latter trials were taken as measures of similarity with the reasoning that the more similar two different scenes are, the harder it would be to respond “different.” In the short deadline speeded judgments, MIPs and MOPs had similar influence on errors. As the deadline became longer, MIPs grew in importance over MOPs.

These concur with predictions by SIAM. As SIAM executes more cycles of activation adjustment, feature-to-feature nodes become increasingly influenced by object correspondences. At first, how strongly two features correspond to each other depends mostly on the features’ similarity; two features tend to be put into correspondence if they are identical or highly similar. With more time, feature correspondences depend increasingly on object correspondences. Specifically, features tend to be placed in strong correspondence if they belong to object that are in correspondence. In turn, objects initially tend to be put into correspondence if they are similar to each other. After several cycles have passed, object correspondences reflect the objects’ feature similarity and consistency with other object correspondences, and feed activation back down to the feature correspondences with which they are consistent.

Each of the studies demonstrates the mutual influence of representations during comparisons. Features are selectively weighted based on influences exerted by the dynamic interplay of representations.

Additional Empirical Evidence for Flexible Weighting

Providing additional support for flexible weighting of features during comparisons, Goldstone, Medin & Halberstadt (1997) found evidence of intransitivities in forced-choice similarity judgments. In a forced-choice similarity task, participants are asked to judge which stimulus out of a set is closest to a standard stimulus. Models of forced-choice similarity judgments that assume predetermined similarities between objects must predict that if stimulus A is more similar to a standard stimulus T than stimulus B, and B is more similar to T than C, then A will be more similar to T than C. Stimuli were wedge-like shapes of varying hues and sizes. A standard stimulus was shown at the top of the screen, and two alternatives were shown below. Subjects indicated which of the two was most similar to the standard. For example, the shapes in Figure 4 were shown in three trials: A paired with B, B paired with C, and A paired with C. A model with fixed similarities predicts that participants will choose one alternative consistently over the others, or (if the stimuli are nearly similar) predicts participants will choose randomly. The majority of responses indicated intransitivities, with B > A, C > B, but A > C, where B > A indicates the B was closer to the standard. This is contrary to how a “diagnostic-dimension” model would be expected, that the influence of the diagnostic dimension that most heavily contributed to the similarity judgment were given the greater weight. The results show that the influence of its diagnostic dimension depends on its diagnostic status, presenting additional evidence for dynamic processes. With this flexible weighting of features, there is no intrinsic basis for intransitivities from additional processing time.

Dynamic Default Theory

Tversky (1977) proposes a default similarity metric that is responsible for determining how two objects are similar. Representations
In these sample stimuli, color differences are represented by shading. In the most responses, B was judged to be more similar to the standard than A(B > A), C was more similar to the standard than C(C > B), but A was judged more similar to the standard then C(A > C).

cates the B was chosen to be more similar than A to the standard. This ordering is consistent with a “diagnostic-dimension strategy,” where the dimension that most differentiates the two choices is given the greatest weight. This result suggests that the influence of a dimension on similarity depends on its diagnosticity within a set of alternatives, presenting an additional challenge to models of similarity. Multiple comparisons in a single task may influence one another; comparisons in such situations must be modeled as interactive processes. While SIAM dynamically adjusts weighting of features within a single comparison, there is no intrinsic mechanism to handle influences from additional comparisons.

Dynamic Development of Features Theory

Tversky (1977) posits that preprocessing is responsible for determining which features are to be considered for a particular similarity judgment. Representations of objects include all possible features that might be employed in a comparison. Comparison proceeds after preprocessing has produced a working set of features. Other models of similarity require similar restrictions. In MDS models, although dimensions may be stretched, dimensions are typically not added or altered during comparison. In SIAM, the descriptions of the scenes to be compared are fixed a priori; the roles of objects within scenes and the feature slots and values for those objects are established before processing begins. Although all features for all objects must be fully specified before processing begins, it is possible for SIAM to dynamically change the contribution of particular features to the similarity judgment. SIAM does not, however, allow for the emergence of new features during comparison. One might make the case that SIAM could account for this phenomena by introducing features with zero starting weights. As processing proceeded, these weights could grow, and would appear to be created during comparison. Evidence discussed
below, however, suggests that the number of necessary features would be prohibitively high.

With well-defined and unambiguous stimuli, the assumption that representations are determined a-priori does not introduce complications. Problems arise, however, in cases where the representations of stimuli are not as clear-cut. Interpretation of children's drawings, for example, pose a particular challenge for parents and teachers. Often, comparison of several examples of a child's drawings is the key to discovery of important features. Features may not be established before the comparison process begins, but result from the comparison itself.

Evidence

Ohnishi (1994) showed that features used to compute similarity depend on experimental context. Training on a puzzle causes subsequent similarity judgments to be based on solution distance—a feature not considered by untrained participants. Working with ambiguous figures, Medin, Goldstone, & Cestnik (1993) also demonstrated that features used in similarity judgments could be context-specific where the context could be as simple as the comparison itself. Ambiguous stimuli were interpreted as having properties in common with unambiguous stimuli to which they were compared. Participants were asked to list the features shared by pairs of objects (examples are shown in Figure 5). On a trial, Stimulus B was shown either with Stimulus A or with Stimulus C. When paired with Stimulus A, Stimulus B was described by more than half of the participants as having the feature of 3 protrusions (e.g. "prongs," "fingers," etc.): when asked to list differences A and B, participants tended to report that they differed in their shapes. When shown with Stimulus C, Stimulus B was described by more than half of the participants as possessing 4 protrusions; subjects tended to list as a difference between B and C that B had one short prong. The reported features of Stimulus B were dependent upon the context provided by the comparison itself, and could result in mutually exclusive interpretations. If we hold that all of the features existed prior to the comparison, then we must say that mutually exclusive features exist as descriptions for a particular stimulus. A more plausible interpretation is that the features did not exist before the comparison, and that context necessary to generate features can be as minimal as the objects being compared.

A recent study provides additional support for feature development during comparison processes. Goldstone, Medin, & Halberstadt (1997) showed context effects on similarity ratings of stimuli with dimensions that are naturally backgrounded. A dimension is backgrounded if it is not psychologically registered; for example, in Figure 6, the thickness of the line of the figures is not registered until stimuli with different thickness are compared. Dimensions are backgrounded when there is no variation on them. On seeing a circle drawn in black ink in the center of a card, one often does not imagine that it might have been colored differently, moved to a corner, printed with thicker lines, or drawn three-dimensionally, unless these variations are explicitly mentioned in a “inferred set” describing a stimulus.

Background dimensions, introduced, the direction of comparison. For example, in a condition in orientation and a condition not described, were created in which to be noticed, or for other conditions. In the experiments shown all three stimuli (in any way) or two trials in a trial, A and B were
explicitly mentioned. People naturally created an "inferred set" of possible alternatives when describing a stimulus.

Background dimensions are not noted by the observer, and do not enter into the comparison. Introducing stimuli that vary along a previously backgrounded dimension foregrounds the dimension, adding a dimension on which stimuli are compared. In the experiment, stimuli were created in triplets (A, B and C) and varied along two dimensions. Stimuli A and B varied along one dimension, while C varied along an additional dimension. The additional dimension in C was designed to be backgrounded-until variation was introduced, the dimension would not enter into a comparison. For example, in Figure 6, A and B differ in the orientation of the object. C differs in orientation and in line thickness. In a control condition not described here, additional stimuli were created in which all dimension were likely to be noticed, or foregrounded, in all comparison conditions.

In the experiment, participants were either shown all three stimuli in a single trial (three-way) or two trials (two-way). In the three-way trial, A and B were shown at the top of the display, and B and C were shown on the bottom of the display. In the two-way trial, either A and B, or B and C were shown. Subjects gave similarity ratings for each pair of objects.

In Figure 6, a multidimensional representation of the stimulus is given. If the vertical dimension is a normally backgrounded dimension such as line thickness, then participants who are only given A and B to compare may not even consider their similarity on this dimension when evaluating their similarity. However, when given the comparison between A and C, a second group of participants may consider the vertical dimension because of variation along it between the compared items, and increase their similarity estimates accordingly (reasoning, "A and C may be far on the horizontal dimension, but they are quite close on the vertical dimension, so I will give them an intermediate similarity rating").

In the three-way condition, A-B similarity ratings were significantly higher than the ratings for the A-C pairings remove. In the two-way condition, the high similarity between A and B on the dimension of line thickness was not registered; the dimension was backgrounded. In the three-way condition, variation on the ad-
ditional dimension (e.g. line thickness) caused the dimension to be foregrounded. The effect of backgrounding was strong enough to produce nonmonotonicity-situations in which adding a difference between two objects increases their similarity.

Nonmononoticities are difficult to explain if we assume the same dimensions are being used for all comparisons. Increasing distance along a dimension would lead to lower similarity. Nonmonotonicities are predicted if different dimensions are being considered when A and B are being compared than when A and C are compared. A reasonable interpretation of these results is that dimensions were not fixed before the comparison process: the context created by the comparisons themselves brought into consideration dimensions that did not play a part in comparisons of the same stimuli presented in a different fashion.

In an earlier study, Medin, Goldstone, & Gentner (1993) argued different comparison standards are created depending on the items that are present on a particular trail. In one experiment, participants rated the similarity of pairs of words that were either presented separately ("separated context") or simultaneously ("combined context"). The pairs of words were either related antonymically (e.g. white and black) or by association/category (e.g. skin and hair). Words that were related antonymically received higher similarity ratings than the other words, but only when the words were presented in the separated context. For example, in the separated context, the group of participants that saw the sunrise-sunbeam comparison gave higher similarity ratings than did participants who saw the sunrise-sunset comparison, but this trend was reversed when sunbeam and sunset were simultaneously compared to sunrise for a participant in the combined context group. Medin et al. (1993) argued that the most salient standard of comparison for antonyms in isolation is their dimensional difference (e.g. time of day for sunrise-sunset), which is quite large because they occupy opposite poles. However, when other terms are considered simultaneously, the standard of comparison is enlarged to include the many features shared by antonyms.

Models

Although there are no widely-discussed models of similarity employing dynamic representations, several recent models of analogy do. As previously observed, models of analogy have inspired models of similarity, and may provide insight into the modeling of dynamic feature formation and flexible representations. Models of analogy and similarity can both be said to employ comparison processes. French (1995) makes the case that similarity and analogy should be considered as one and the same. One obvious difference to be noted, however, is analog retrieval. While most models of analogy must concern themselves with processes which search for and retrieve appropriate analogs, models of similarity may assume that the items to be compared are predetermined and readily available. In French's conceptualization of analogy, discussed later, even this difference is minimized.

Hummel & Holyoak (1997) propose a model which partially addresses the problem of flexible representations in models of analogy. In Hummel and Holyoak's Learning and Inference with Schemas and Analogies (LISA) model, analogs are represented as distributed activations of semantic units. The model maintains structured representations by employing dynamic binding, implemented in the model by synchronized firings of associated nodes. The collection of synchronized nodes is termed a group. Since bindings are dynamic, representations of analogs are not fixed. Representation can be altered while in working memory, and then stored in long-term memory. There is a limit to the number of unique groups that can be accommodated at one time. Studies of animal neural systems which seem to employ this type of dynamic binding (e.g. feline visual systems) use 4-6 groups.

In SME, mapping source and target requires that the number of processing steps cannot be more than one (book A, then mapped to the analog B). In LISA, there is no requirement.

In Robert French's Tabletop, representations are organized as processes and not include a means of mapping representations, as do Tabletop operates on a tabletop with knives, forks, and two settings opposite each other. Tabletop's task is to place objects. In a particular setting, one setting is "to touch" the correct setting. For example, a knife and a spoon on one side and a knife and a fork on the other, touching the other operates probabilistically on respondees in the Tabletop pointing, due to failure in generation, to be thought of as more like two sets of "touch" the correct setting.

The architecture of Copycat (Hodges, 1984), but also bears some similarity to Holyoak's LISA. Two components: the Slipper for the Coderack, a Workspace, where words occur. Codelets are limited to a processing neighbor, and problem Codelets are deployed.
visual systems) suggest that a reasonable limit is 4-6 groups.

In SME, mappings between predicates in source and target analogs must be one-to-one. If the number of predicates do not match, a mapping cannot be made. For example, the analog stacked (book A, book B, book C) cannot be mapped to the analog on top of (book A, book B). In LISA, there is no one-to-one mapping requirement.

In Robert French’s model of analogy (1995), Tabletop, representations are generated and re-organized as processing proceeds. Tabletop does not include a memory for previously-generated representations, and so avoids analog retrieval. Tabletop operates in a microdomain of a table top with knives, spoons, cups, etc. There are two settings opposite to each other on the table. Tabletop’s task is to determine corresponding objects. In a particular configuration, an object at one setting is “touched.” Tabletop endeavors to “touch” the corresponding object at the other setting. For example, given a setting of a glass and a spoon on one side, and a cup and a fork on the other, touching the spoon would lead to Tabletop touching the fork. Although Tabletop operates probabilistically, the weight of the correspondences in this case would usually lead to Tabletop pointing to the fork. The success or failure in generating these correspondences may be thought of as a measure of similarity. The more alike two settings are, the easier it is to touch “touch” the correct object.

The architecture of Tabletop is most similar to Copycat (Hofstadter, 1995; Mitchell, 1990), but also bears some resemblance to Hummel and Holyoak’s LISA. Tabletop consists of three components: the Slipnet, a flexible semantic network; the Coderack, a collection of codelets; and the Workspace, where processing of the stimuli occurs. Codelets are small programs which perform limited processing on the stimuli, such as “find neighbor,” and pass activation to the Slipnet. Codelets are deployed probabilistically by activation from the Slipnet. Both LISA and Tabletop use activations distributed across a semantic network; the Slipnet, however, is not a connectionist network.

Representations of the configuration of objects on the table is built up by the activity of the codelets. Groupings and correspondences that are promising receive greater attention from codelets. Representations can be discarded if they are not contributing to a solution. Tabletop demonstrates both top-down and bottom-up influences, and flexible representations. For example, giving the setting of spoon-fork-spoon on one side of the table and knife-sah-knife on the other. Tabletop initially does not notice that there is a group of spoons. Codelets may cause this group to be formed, allowing for the interpretation of the knives as a group, creating a new feature used to describe the setting.

Uncovering Dynamic Processes

The models discussed above demonstrate various levels of representational flexibility (Table 1). Feature contrast models assume fixed sets of features, but allow differential weighting of features shared by the compared items and features unique to each item. Standard MDS models assume the least flexible representations, with all dimensions fixed before processing. In the Generalized Context Model, dimensions exist before comparison, but the weighting of a dimension is determined by the global context. For example, if size is important in an ongoing categorization, the weighting of the size dimension increases reflecting the greater attention given to size. In SIAM, the influence of a feature changes during comparisons. SME allows features to be carried over from a source analog to a target analog if a structural match is found between the analogs. LISA relaxes the restrictions for a structural match between analogs. Copycat and Tabletop dynamically create and discard interpretations of scenes.

Studies discussed earlier provide evidence of
items exercising mutual influence during comparison processes: flexible weighting of features in similarity judgments, and mutual development of features during comparisons. The specifics of the psychological dynamics of comparisons are less well-known. Questions remain regarding the time course and degree of mutual influence of items during comparisons.

These questions can be phrased in terms of processing dynamics. Items that do not influence one another during comparisons can be said to be processed separately. Items that mutually influence one another during comparisons exhibit different processing behavior. Insight into the processing taking place during comparisons gives information about the types of representations used in that processing. Townsend & Nozawa (1995) provide a methodology which may be appropriate for differentiating these types of processing.

The "double factorial" paradigm provides methods for determining system architecture and capacity within a single experimental design, where capacity is defined as a measure of the influence that varying load has on performance. For example, one may vary load by increasing the number of items to be processed. Capacity is a measure of the ability of a system to handle increased load. Simply put, fixed capacity refers to a system that is increasingly influenced (i.e., slowing down, producing errors) as the number of items to be processed increases. Unlimited capacity refers to a system that is not so influenced. Supercapacity refers to a system that benefits (i.e., speeds up, produces fewer errors) as load increases (Townsend & Ashby, 1983). The double factorial paradigm differentiates serial, parallel, and coactive architectures with different capacities. In serial architectures, items are processed one at a time. In parallel architectures, multiple items are processed simultaneously but separately. In coactive architectures, multiple items are processed simultaneously and processing is not separated. Coactive architectures can take many forms; in some types of coactive architectures, items may mutually influence one another as they are being processed. Townsend and Nozawa's methods draw from the entire reaction time distribution and provide distinct tests for architecture and capacity.

Townsend & Nozawa (1995) applied this technique in a psychophysical experiment; the method, however, can be applied to problems in analog and similarity. A simple word analogy will serve as an example. Given a task of determining a relationship between word pairs, a participant is presented with two types of trials. In the single presentation trial, a single word pair is shown such as "embellish : austere." In a double presentation, two word pairs are shown such as "embellish : austere" and "adulate : pure" that express similar relationships. The participant is to respond as soon as any relationship becomes clear. Controlling for reading time, one may ask if the presentation of the additional word pair will speed or slow the process of comprehension.

The participant may be capable of processing only one relationship at a time, may process two word pairs in a single trial independently, or may benefit from having two word pairs. In the first case, one can expect slower processing in
the double presentation trial when compared to the single presentation trial. This reflects a finding of limited capacity. In the second case, however, one can anticipate a simple statistical advantage in presenting two word pairs. Processing is speeded since both word pairs are "racing" to a solution. Since the participant can respond as soon as one relationship becomes clear, the double presentation trials will be faster when compared to the single presentation trials. This reflects a finding of unlimited capacity. If the speed advantage is greater than that predicted in the "race," processing is supercapacity. A finding of supercapacity is consistent with coactivation; information from each word pair may serve to help build the relationship in the other, leading to a faster solution.

Reaction time distributions are used to calculate capacity. To produce reaction time distributions, one first constructs a graph of frequencies of reaction times, as in Figure 7 (A). One then converts this distribution into a cumulative dis-
turbation, as in Figure (B). At this point, using the cumulative distribution of the single presentation trials, one can predict what performance should be in the double presentation case if the system is unlimited capacity parallel processing (the “race” described above). If the double presentation exceeds the predictions, there is evidence for supercapacity and coactivation. In the example, there is such a violation.

The Townsend and Nozawa (1995) measure of capacity is the capacity coefficient:

$$C(t) = \frac{H_{LR}(t)}{H_L(t) + H_R(t)}$$

where $H_{LR}(t)$ is the integrated hazard function of the double presentation reaction time distribution at time $t$, and $H_L(t)$ and $H_R(t)$ are the integrated hazard functions of the single presentation reaction time distribution at time $t$. For this example, $H_L(t)$ and $H_R(t)$ are identical. The hazard function can be interpreted as the probability that a process will complete given that it hasn’t completed yet. In this example, a process completes when a relationship is found in a word pair. $C(t)$ that is less than 1 implies limited capacity. $C(t)$ that is near 1 implies unlimited capacity, as is the case condition. $C(t)$ that is greater than 1 implies supercapacity and coactivation.

Coactivation and supercapacity in similarity and analogy processes should be anticipated for flexible representations. Dynamic weighting and dynamic feature development in similarity processes suggest that processing does not proceed in an independent manner. Supercapacity may result when representations of items are complementary. For example, comparisons of scenes made up of objects with both ambiguous features (such as those shown in Figure 5) and unambiguous features can result in complementary representation building. Objects with unambiguous features can provide context for quick interpretation of corresponding objects with ambiguous features. If comparison proceeds in an independent, parallel manner, each feature must be disambiguated before processing can progress. If comparison proceeds in an interactive manner, objects which are found to correspond provide context for interpretation of ambiguous features. The capacity measure can provide evidence concerning the critical question of whether representations of entities in comparisons are shaped in response to one another, and under what circumstances we can expect this mutual representation building. Similarly, in the Tabletop model, interpretations of groups of objects in a setting cause reinterpretation of groups at the other setting. The settings inform one another, building representations that might not have evolved from examining a single setting alone. Models of comparison that incorporate representation building could be expected to show supercapacity; increasing the number of objects could serve to speed solution. Comparisons of simple or well-defined entities that do not invoke flexible representation could be expected to exhibit the capacity of a parallel processing system.

Summary

Comparison processes in similarity and analogy can lead to emphasis of particular features and to the realization of new feature descriptions. Similarity and analogy involve dynamic processes in which representations are flexible, changing over time, and changing in response to context. Presented individually, Monet’s paintings of the Houses of Parliament are studies of a building under various environmental conditions. Presented together, the impact of the paintings changes: the major feature becomes the envelope of light surrounding the buildings, the time and changing in response to context. Presented together, the impact of the paintings changes: the major feature becomes the envelope of light surrounding the buildings, the time the buildings themselves seem to interact with the environment. Objects undergoing comparison build upon one another, like two rising towers, each supporting the other and reaching heights not achievable alone.
References


(1997年6月23日受理)
(1997年11月10日採録)

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The Metaphor A Self-Writing

1. Introduction

This paper describes an extension of the Copycat model, a case-based learning system, to high-level analogical reasoning that accounts for the conscious interplay between analogical and logical inferences. Following Mitchell (1993), Copycat operates in a crowsworld of analogies, propositions, and concepts. The crowsworld admits a wide range of analogical mappings, and every imaginable crowsworld admits a crowsworld of analogies.

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This paper was presented at the Annual Meeting of the Cognitive Science Society, University of Oregon, June 1997.