Effects of Grounded and Formal Representations on Combinatorics Learning

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Abstract

Two experiments examined the differential effects of grounded and formal representations on learning of mathematics. Both involved combinatorics, using outcome listing and combinatorics formulas as examples of grounded and formal representations, respectively. Experiment 1 compared performance on near and far transfer problems following instructions involving listing or formulas. Instruction in formulas led to more near transfer, while far transfer performance did not differ by condition. Experiment 2 compared performance following four types of instruction: listing only, formulas only, listing fading (listing followed by formulas), and listing introduction (formulas followed by listing). The listing fading condition led to performance on par with the formulas only condition, and for near transfer problems, significantly higher than the listing introduction and pure listing conditions. The results support the inclusion of grounded representations in combinatorics instruction, and suggest that such representations should precede rather than follow formal representations in the instructional sequence.

Keywords: mathematics; formalisms; grounded representations; transfer; analogy; education

Background

Alternate Representations in Mathematics

Mathematical ideas often admit of alternate representations. Much research has investigated the differential effects of mathematics instruction based on formal representations, such as equations, or more grounded representations, such as diagrams. Formal representations such as algebraic equations have been found, in some contexts, to promote learning and transfer better than grounded representations. One possible reason is that idealized or abstract representations may better draw attention to underlying logical structure, while perceptually rich representations distract from it (Sloutsky, Kaminski, & Heckler, 2005). There is also evidence that using concrete problems to learn mathematical concepts may inhibit transfer (Bassok & Holyoak, 1989).

Additionally, it is possible that problems represented in abstract symbolic form are simply easier to solve than those, such as story problems, that refer to concrete entities. This view seems prevalent among educators: in one survey of primary and secondary mathematics teachers, a majority believed that their students found story problems more challenging than mathematically isomorphic equation problems (Nathan, Long, & Alibali, 2002). The same belief is reflected in the equations-before-story problems sequence prevalent in mathematics textbooks. The rationale seems to be that story problems must be converted into equations in order to solve them, making equation problems *a priori* easier.

In reality, however, primary and secondary school students perform better on simple story problems than on mathematically equivalent equation problems (Koedinger & Nathan, 2004), while the reverse trend obtains only for more complex problems (Koedinger et al, 2008). Story problems seem to encourage the use of certain intuitions and informal solution strategies that, relative to standard algebraic procedures, lead to greater success on simpler problems. Algebraic procedures lead to greater success on more complex problems for which informal strategies are less feasible. In this domain, neither grounded nor formal representations are simply preferable to the other; each has its own strengths.

If simpler problems are facilitated by grounded, and complex problems by formal, representations, then beginning with grounded representations and proceeding to more formal representations may be a sound pedagogic strategy. Such an approach has been advocated by Freudenthal (1991) and also derives support from research on "concreteness fading," in which learners are exposed first to concrete instances of concepts, and later to more idealized representations. McNeil and Fyfe (2010) trained students on the idea of modular arithmetic using either concrete, idealized, or concrete followed by idealized, representations. Students in the last condition showed the best performance on novel transfer problems. Similar benefits of concreteness fading have been shown for understanding of complex systems principles (Goldstone & Son, 2005).

The Combinatorics Domain

The present study uses the domain of combinatorics as a testing ground to examine the differential effects of instruction using formal and grounded representations on learning and transfer. From a pedagogic standpoint, combinatorics plays an important role both in mathematics education and in education more generally. In mathematics education, combinatorics is fundamental to the theory of probability and statistics, which has a wide range of practical applications. More generally, insofar as combinatorics requires a systematic consideration of what is possible, independent of what actually is, its mastery is considered to be one step in in the general development of abstract reasoning capabilities (Inhelder & Piaget, 1958).

Figure 1 shows an example of one type of combinatorics problem: sampling with replacement (SWR). SWR problems may be solved by using the formula m^n, where m is the number of items in the set being sampled, and n is the number of times sampling occurs (Figure 1a). In addition to such formal expressions, mathematics students often employ a range of more grounded visual representations to solve such problems (Corter & Zahner, 2007). One such representation is outcome listing (Figure 1b). A complete list of outcomes may be generated through a systematic strategy such as the "odometer" strategy, which involves exhaustively varying the outcome for a single sampling event while holding the outcomes of all the other sampling events constant. Another category of combinatorics problems is permutations (PER) problems. PER problems, like SWR problems, admit of solution either by a formula – *m!*, where *m* is the number of items being permuted – or by a systematic listing strategy.

A marketing research company asks people to taste three different brands' pizza (Domino's, Edwardo's, and Pizza Hut) and then asks which one they liked the best. If the company surveys TWO people, how many different results are possible?

(a) $3^2 = 3 \times 3 = 9$

(b)

D D	E D	P D
D E	E E	P E
D P	E P	P P

Figure 1. A combinatorics problem.

The distinction between standard combinatorics formulas and outcome listing corresponds to the more general distinction between formal and grounded representations in mathematics. Clearly, combinatorics formulas constitute formal representations. By contrast, lists of possibilities are more grounded than formulas, because the former involve actual numerosities, the latter only number symbols – for example, where the formulas use the numeral 3, the lists actually show three different letters. (The fact that letters are also symbols does not detract from the general point that outcome lists represent number in a more grounded way than do combinatorics formulas.) The present study explores the effects of instruction employing these alternate representations on learning and transfer.

Experiment 1

In this experiment, participants were shown worked examples of combinatorics story problems belonging to one category – either SWR or PER. Subsequent performance on novel problems of the *same* category was used as a measure of "near transfer," while subsequent performance on the *other* category was used as a measure of "far transfer." Near transfer, thus defined, is not trivial: even if the transfer problems belong to the same category used during instruction, differences in their "stories" can make the transfer problems challenging. Far transfer, thus defined, is still more difficult: it requires participants not only to navigate differences between the stories of the worked examples and those of the transfer problems, but also to derive entirely novel solution methods, presumably by adapting the methods shown during instruction. Such adaptation might be possible due to the structural similarities between the two problem categories (Figure 2). The formal solutions for both categories involve multiplying a sequence of numbers beginning with the number of elements in the set from which selections are made, with the total number of multiplications equal to the number of elements selected.

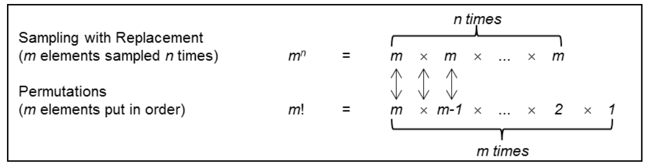

Figure 2. Correspondence between SWR and PER formulas.

This experiment was designed to investigate the differential effects of formula- and listing-based instruction on near and far transfer performance. Insofar as combinatorics formulas make explicit the mathematical structure common to all problems of the same category, while outcome listing does not, we might expect instruction in formulas to result in more near transfer than instruction in outcome listing. As for far transfer, however, formal instruction might fare less well. It is not at all evident how to derive the PER formula from the SWR formula or vice versa. Adaptation of the corresponding listing procedures may prove easier for learners. Many aspects of a systematic listing strategy apply equally well to either problem type, and any adaptation required may be relatively intuitive based on the common everyday experience of arranging physical objects in sequence.

This experiment also tested a secondary prediction regarding the effects of formula- and listing-based instruction on problems of varying degrees of complexity. Koedinger et al's (2008) results suggest that formal solution methods might show an advantage on relatively complex combinatorics problems. By contrast, the more intuitive approach of listing outcomes might be more effective for simpler problems. In sum, formula-based instruction was predicted to lead to better performance on near transfer and complex problems, while listing-based instruction was predicted to show an advantage on far transfer and simple problems.

Materials and Methods

Participants. 126 undergraduate and graduate students from Indiana University participated in the experiment, including 78 students who participated for course credit, and 48 students who participated for a financial incentive.

Materials. Two sets of combinatorics story problems were developed for testing participants. Each set consisted of four problems: two SWR and two PER, with one "simple" and one "complex" problem for each category. The complex problems required solution of three simple sub-problems followed by summation of their solutions. For example, finding how many sequences of the notes C, E, and G are possible that are 5 notes long (answer: 3^5=243) constitutes a simple SWR problem, while finding how many such sequences are possible that are 3, 4, or 5 notes long (answer: $3^3+3^4+3^5$=351) constitutes a complex SWR problem. The two test problem sets were mathematically isomorphic, but used different cover stories.

In addition to the test problem sets, several training sequences were developed. Each sequence consisted of three story problems, all belonging to the same category, and a Powerpoint slideshow presenting worked solutions. The sequences differed in terms of problem category – SWR or PER – and method used in the worked solutions – either combinatorics formulas or outcome listing, as described in the Background. For a given problem category, the same set of problems were used for the formula and listing versions. There were thus four training sequences altogether: one for each possible combination of category and solution method.

Procedure. The study employed a pretest – training – posttest design. Participants were given paper questionnaires consisting of one of the test problem sets, referred to as "pretest," one training problem set, and the other test problem set, referred to as "posttest." Which test problem set was used as pretest and which as posttest, and which problem category was used for the training problem set, were assigned randomly. Participants were randomly assigned to view either the formula or the listing version of the worked solutions to their training problem set, and worked in front of computers containing the appropriate Powerpoint slideshows. The type of solution method viewed constitutes the principal between-subjects variable of the study, and is henceforth referred to as "training condition."

Participants were asked to solve the problems in order and not to return to any problems after completing them. They were encouraged to show their work as much as possible. Participants were instructed to view the slideshows on the computers when directed to do so by the paper questionnaire (i.e. when solving the corresponding training problems).

Coding. Pretest and posttest responses were classified as either correct or incorrect. Numeric expressions that evaluated to the correct answer, such as "3×3×3×3×3" for "243", were accepted as correct. Correct answers were assigned a value of 1, and incorrect answers a value of 0. A transfer score was calculated for each of the four test problem types by subtracting the scores for the pretest problems from those for the corresponding posttest problems. Thus, each participant received a transfer score for each problem type ranging from -1 (decrement) to 0 (no change) to 1 (improvement).

Transfer performance data was re-categorized according to transfer distance without regard to problem category. In other words, the data for PER problems were classified as near transfer for participants trained on PER and as far transfer for those trained on SWR, and vice versa for SWR problems. Thus, each participant received four transfer scores: one for each combination of transfer distance (near or far) and problem complexity (simple or complex).

Results and Discussion

Results. Mean transfer performance data is shown in Figure 3. One-sample two-tailed t-tests conducted for each problem type found that transfer performance was significantly greater than 0 (with the criterion α=.05) for both near transfer problems, but not for either far transfer problem. Within each training condition, improvement was significant in the formula condition for simple near transfer only, and in the listing condition for complex near transfer only.

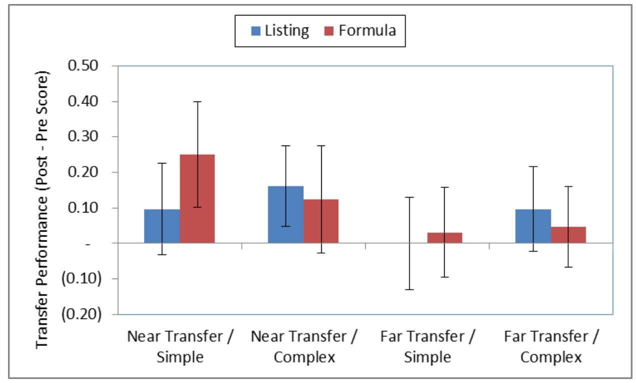

Figure 3. Mean Transfer Performance.

The data were entered into a linear mixed model, with performance change as the dependent variable, transfer distance and problem complexity as within-subjects variables, training condition as a between-subjects variable, and pretest score as a covariate. There was a significant effect of distance, indicating more improvement for near transfer (0.16) than for far transfer (0.04), F(1,361.1)=24.6, p<.001. The main effects of problem complexity and training condition were not significant. However, there was a significant interaction between distance and condition, F(1,360.3)=4.5, p=.035, reflecting an advantage of the formula condition on near transfer (formula: 0.19, listing: 0.13) but little difference between conditions on far transfer (formula: 0.04, listing: 0.05). No other interactions reached significance.

Discussion. For near transfer problems, both formula and listing instruction resulted in significant posttest improvement, and the observed interaction effect suggests a relative advantage for formula instruction, consistent with our predictions. For far transfer problems, significant posttest improvement was not observed in either condition. This apparent floor effect precludes any claim as to the superiority of either type of instruction for promoting far transfer.

Aside from the sheer difficulty of the far transfer problems – an issue addressed in the next experiment – there are several possible explanations for why listing training failed to show the predicted advantage over formula training. First, it is possible that participants actively resisted the outcome listing approach. Although some participants in the listing condition did produce outcome lists on posttest, many did not, preferring to use purely numerical calculations. One such participant commented that she would have preferred simply being told how to do the problems – that is, how to solve them with formulas – reflecting a belief that outcome listing was not a "real" solution method. Such a belief might relate to the greater efficiency of formulas, or to a greater emphasis on formulas in previous education.

Second, participants in the listing condition may have been impeded by the need to integrate lists with numerical calculations. The test problems (though not the training problems) involved numbers sufficiently large that solution by outcome listing *alone* was not feasible in the time provided. Use of outcome listing would require creation of a partial list followed by some form of numerical operation, such as multiplication of the partial list size by the number of partial lists that would occur in a complete list. This additional step might be challenging, either simply by virtue of being an additional step, or because it requires integration of two very different modes of thought: grounded and formal.

Experiment 2

Although it is useful to know how instruction based on formalisms alone compares to instruction using only grounded representations, actual classroom instruction often involves a mixture of both types. Some existing research has supported this approach, showing a learning advantage for instruction involving both concrete and idealized representations over instruction involving only one or the other (Goldstone & Son, 2005; McNeil & Fyfe, 2010). Goldstone and Son (2005) additionally found effects of order: "concreteness fading," i.e. beginning with concrete representations and proceeding to idealized ones, worked better than the reverse sequence, "concreteness introduction." Van Reeuwijk (1995) employed a similar "progressive formalization" approach, beginning with grounded representations and proceeding to algebraic formalisms.

Experiment 2 explored the effectiveness of such approaches in the context of combinatorics, using the same instances of grounded and formal representations as in Experiment 1: outcome lists and combinatorics formulas. Two specific hypotheses were suggested by the above-mentioned literature on concreteness fading and progressive formalization. First, better transfer performance was predicted after instruction incorporating both lists and formulas than after instruction employing only one or the other. Second, for instruction employing both lists and formulas, better performance was expected when lists were introduced before formulas rather than after.

Materials and Methods

Participants. 111 undergraduate students from Indiana University participated in the experiment for course credit.

Materials. Like Experiment 1, this experiment employed two test problem sets and a training sequence. All four test problems belonged to the PER category and involved the same cover stories used for this category in Experiment 1. The first two problems were categorized as "near transfer" because they could be solved by direct application of the solution method shown during training. The first of these was mathematically isomorphic to one of the training problems, while the second was identical to the "simple" PER problem used in Experiment 1. The next two problems were categorized as "far transfer" because they required some adaptation of the solution method shown in training. The first of these was identical to the "complex" PER problem used in Experiment 1, while the second was a novel problem requiring permutation of a partial subset – a less "distant" far transfer problem than that used in Experiment 1.

As in Experiment 1, the training sequences consisted of combinatorics story problems accompanied by Powerpoint slideshows. The sequences involved four story problems, all belonging to the PER category, the first two using one cover story and the second two using a different cover story. There were four versions of the accompanying slideshows. (1) The pure listing version demonstrated solution of all four problems by systematic listing of possible outcomes. (2) The pure formula version demonstrated solution by numerical computation. (3) The listing fading version used outcome lists for the first two problems and formulas for the latter two. (4) The listing introduction version employed the same content as in (3), but in the reverse sequence. Both formula and listing solution methods were presented in a slightly different way from that in Experiment 1.

Procedure. The study employed a pretest – training – posttest design similar to that used in Experiment 1, with two important differences. First, SWR problems were not used, so all participants received only PER problems in both the test and training problem sets. Second, the internal sequence of training problems was rotated among participants, independently of the type of training received, by randomly assigning which pair of problems came first and which second. The method of data collection was also similar to that used in Experiment 1, with two important differences. First, all problems were presented via computer, and participants were asked to show their work and enter their answers directly into the computer. Second, participants were allowed to use calculators, which were shown on the computer screen beside the experiment interface.

Coding. Pretest performance, posttest performance, and transfer performance were calculated for each test problem in the same way as for Experiment 1. The data from the first two test problems were combined to derive aggregate scores for near transfer, and those from the second two problems to derive scores for far transfer. Additionally, participants' shown work for each problem was assigned one or more codes according to the solution method(s) used. The analysis presented here concerns only two of the codes employed for this task: "numerical calculation" and "outcome listing."

Results and Discussion

Results. Mean transfer performance data are shown in Figure 4. One-sample two-tailed t-tests conducted for each transfer distance and training condition found that transfer performance was significantly higher than zero for all conditions except listing introduction for near transfer, and for all conditions except pure listing for far transfer, using the criterion α=.05.

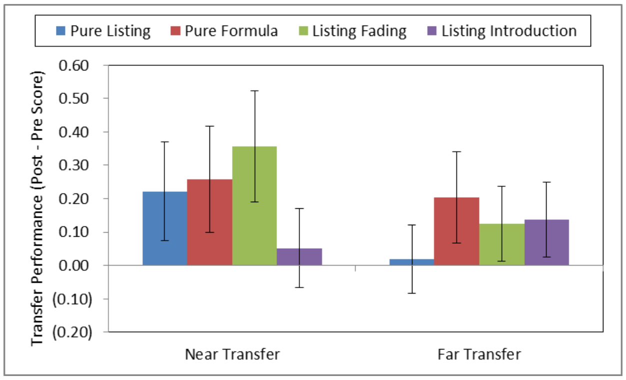

Figure 4: Mean Transfer Performance.

The data were entered into a linear mixed model, with performance change as the dependent variable, transfer distance as a within-subjects variable, training condition as a between-subjects variable, and pretest score as a covariate. There was a significant effect of distance, indicating more improvement for near transfer (0.22) than for far transfer (0.12), $F(1,125.6)=42.2$, $p<.001$. The main effect of training condition was not significant, but there was a significant interaction between distance and condition, $F(1,107.2)=4.3$, $p=.006$. The same model run for near transfer problems only showed a significant effect of training condition, $F(3,106)=2.9$, $p=.04$. Pairwise comparisons between conditions for near transfer showed significantly greater transfer performance in the listing fading (0.36) than the pure listing (0.22) and listing introduction (0.05) conditions, $F(1,52)=4.4$, $p=.040$ and $F(1,54)=6.5$, $p=.013$ respectively. No other pair of conditions differed significantly for near transfer. The same model run for far transfer problems only found no significant effect of training condition.

Finally, the codes assigned to participants' shown work were analyzed to determine whether participants actually used the methods they were instructed to use on the training problems. Participants were considered to have followed instructions if they used the instructed method at least once for both the first and second pairs of training problems. By this standard, participants followed instructions most in the pure formalism condition (100%), followed by listing fading (57%), pure listing (52%), and listing introduction (28%). The difference among conditions was significant, $p<.001$ by Pearson's Chi-Square, and was primarily driven by low usage of outcome listing in the latter three conditions.

Discussion. This experiment was designed to explore the effectiveness of a listing fading approach to combinatorics instruction, in which a grounded representation – outcome lists – precedes a corresponding formal representation – a combinatorics formula. Consistent with our predictions, listing fading led to the highest average transfer performance of the conditions tested, and for near transfer problems, showed a significant advantage not only over pure listing, but also over listing introduction. The latter advantage is striking because listing fading and listing introduction employed the same materials, differing only in the sequence of presentation. These results suggest that listing fading is indeed a viable instructional approach in combinatorics, and are consistent with the general view that "fading" from grounded to formal representations is an effective strategy, especially in comparison to the reverse sequence.

However, this conclusion must be qualified in two respects. First, despite its strong performance, the listing fading condition showed no advantage over the pure formula condition. Thus, the results do not support a strong claim as to the necessity of including outcome listing in combinatorics instruction. Second, for far transfer problems, no significant effect of training condition was found. Thus, the results do not support any claim that listing fading leads to more flexible knowledge and thus greater far transfer than either pure formula or listing introduction instruction.

During training, while participants virtually always followed instructions to use numerical calculations, they often did not use outcome listing when instructed to do so. This apparent resistance to outcome listing may result from simple unfamiliarity, or from a belief that numerical methods are superior and/or more appropriate for problems in this domain. Resistance to grounded representations by students with prior exposure to formal methods has also been found in the domain of algebra equation solving (van Reeuwijk, 1995). Thus, students may not spontaneously reap whatever benefits are to be gained from exposure to grounded representations in combinatorics. Teacher intervention may be crucial to realizing any such benefits.

General Discussion

The two experiments described herein investigated the effects on near and far transfer performance of instruction employing grounded and formal representations in the mathematics of combinatorics. Outcome listing and combinatorics formulas were taken as examples of grounded and formal representations, respectively. Instruction involving formulas only led to rates of near and far transfer equal or superior to the best results produced by instruction involving outcome listing. Other studies of combinatorics learning have also found either no advantage of grounded representations, or even an actual advantage for formulas (e.g. Kolloffel, 2008). Clearly, formulas remain an effective, probably essential, element of combinatorics instruction.

Nevertheless, the present results do suggest that grounded representations such as outcome listing may have potential benefits as well. Instruction employing *only* outcome listing resulted in moderate or poor transfer in both experiments. However, in the listing fading condition of Experiment 2, instruction using *both* listing and formulas resulted in as much transfer as that using formulas only, and more near transfer than that using listing only. It is reasonable to ascribe some positive effect to the listing part of that instruction, because if it had none – if only the formula part was effective – then its effects on transfer should have been inferior rather than equal to those of pure formula instruction, which included twice as much exposure to formulas.

The listing fading condition of Experiment 2 also led to more near transfer than the listing introduction condition. This result is consistent with the general view that introducing grounded representations before, rather than after, formal ones leads to better learning outcomes (Goldstone & Son, 2005; Koedinger et al, 2008). One possible explanation is that grounded representations provide learners with intuitively comprehensible scaffolding on which they can subsequently build formal knowledge. Another explanation, discussed further below, is that learners, if first exposed to formal representations, may perceive grounded representations as irrelevant and consequently ignore them. Of course, these two possibilities are not mutually exclusive.

Instruction involving outcome listing was predicted to promote far transfer more than formulas-only instruction, on the grounds that learners would find lists more intuitive and flexible than formulas. One might also make the same prediction on the basis that, relative to formulas alone, outcome lists should promote greater conceptual understanding, on which far transfer presumably relies (Rittle-Johnson & Alibali, 1999). However, this prediction was not confirmed. Rates of far transfer did not differ by training condition in either of the experiments reported. This negative result might have been caused by a floor effect in Experiment 1, but not in Experiment 2, in which significant far transfer was observed. Outcome listing seems to have conferred no particular advantage for far transfer.

The absence of such an advantage may indicate that outcome listing simply does not, as supposed, conduce to more flexible knowledge or greater conceptual understanding of combinatorics. However, it is also possible that the potential cognitive benefits of outcome listing were diluted by resistance to this representation on the part of some participants. Consistent with this interpretation, participants in Experiment 2 often did not use outcome listing when instructed to do so, especially after prior exposure to formulas. Understanding the degree to which such resistance exists, and the reasons behind it, would be crucial to successful use of outcome listing in combinatorics instruction, and by analogy, of grounded representations in instruction in other areas of mathematics as well.

One possible reason why learners might resist the use of outcome listing in combinatorics is that they perceive it as non-mathematical and irrelevant to the “real” (i.e. formal) solution methods. However, outcome lists must be relevant to combinatorics formulas at least in the sense that the two interact in learners’ minds, as if they did not, it would not matter in what order they were encountered. In Experiment 2, such interaction was relatively uncontrolled: participants in the mixed conditions were simply exposed to both representations in sequence. Instruction that more actively encouraged learners to integrate their knowledge of alternate representations to form coherent conceptual understanding would likely increase the benefits of using both representations. Such integration might be achieved through drawing explicit connections between corresponding elements of alternate representations and / or by practice in translating from each representation to the other. The potential of such methods to increase the benefits of grounded representations to mathematics instruction is likely to be a fruitful direction for further research.

Acknowledgements

This research was in part supported by National Science Foundation REESE grant 0910218.

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